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AP Biology, Athens Academy

Variability, Standard Deviation, and Standard Error of the Mean with a lot of help from our friends at *Wikipedia* and from *A Short Guide to Writing About Biology* by Jan A. Pechenik.*

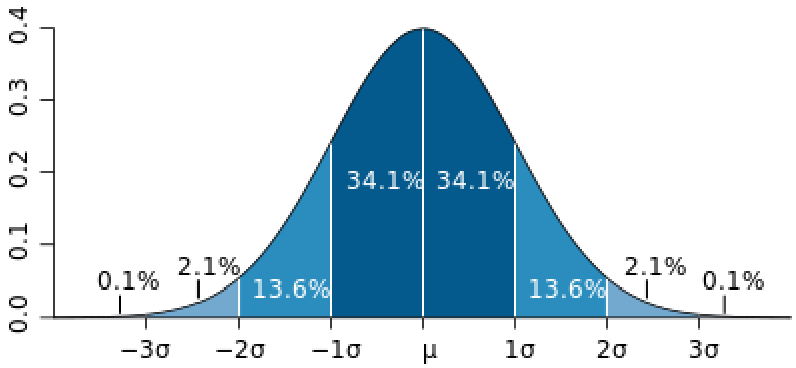
> **Variability is a fact of biological life**: Student performance on any particular examination varies among individual: the growth rate of tomato plants varies among seedlings, and from place to place and year to year, or even week to week; the effects of a particular concentration of a particular pollutant vary among species, and among individuals within a species; the respiration rate of mice held under a given set of environmental conditions varies among individuals; the number of snails occupying a square meter of substrate varies from place to place and from year to year; the extent to which a particular chemical enhances or suppresses the transcription of a particular gene varies from test tube to test tube; and the amount of time a lion spends feeding varies from day to day and from lion to lion. Of course, some of the variability we inevitably see in our data reflects unavoidable imprecision in making measurements. If you measure the length of a single bone 25 times to the nearest millimeter (mm), for example, you will probably not end up with 25 identical measurements. But much of the variability recorded in studies reflects real biological differences among the individuals in the sample population. Put identical meals in from to 20 people in a restaurant, and few of these people will finish their meals at the same time. Moreover, the amount of food consumed will probably also vary quite a lot among individuals. This sort of natural variability is referred to as "error" by statisticians, but it isn't "error" in the sense of "making mistakes." It is better to think of such variation as natural "scatter" in the data.*

There are a couple of descriptive statistics that can be useful in describing the extent of variation we see in an experiment of a set of observations: the standard deviation and the standard error of the mean. Their uses are a little different, but they are both helpful in describing the scatter of the data you collected in your experiments.

In statistics and probability theory, **standard deviation** (represented by the symbol sigma, **σ**) shows how much variation or dispersion (scatter) exists from the average (mean), or expected value. A low standard deviation indicates that the data points tend to be very close to the mean—if all the values were the same there would be no variation and the SD would be 0; high standard deviation indicates that the data points are spread out over a large range of values.

The figure below shows a “normal distribution” of values for a population (a bell-shaped curve). It shows the percent of values that fall within different values of standard deviations from the mean. As you can see, 68.2% of the population falls within one standard deviation (+ or -) of the mean, where as just over 95% of the population will fall within 2 standard deviations from the mean.

The IQ of adults in North America will be distributed in this same manner with a mean of around 100.



Here is how you calculate the standard deviation:

Consider a population consisting of the following eight values:

$$2,\ 4,\ 4,\ 4,\ 5,\ 5,\ 7,\ 9.$$

These eight data points have the **mean** (average) of 5:

$$\frac{2+4+4+4+5+5+7+9}{8} = 5.$$

To calculate the population standard deviation, first compute the difference of each data point from the mean, and square the result of each:

$$\begin{array}{ll} (2-5)^2 = (-3)^2 = 9 & (5-5)^2 = 0^2 = 0 \\ (4-5)^2 = (-1)^2 = 1 & (5-5)^2 = 0^2 = 0 \\ (4-5)^2 = (-1)^2 = 1 & (7-5)^2 = 2^2 = 4 \\ (4-5)^2 = (-1)^2 = 1 & (9-5)^2 = 4^2 = 16. \end{array}$$

Next, compute the average of these values (called the **"variance"**), and take the square root of it:

$$\sqrt{\frac{(9+1+1+1+0+0+4+16)}{8}}=2.$$

This quantity is the *population* standard deviation, and is equal to the square root of the variance. The formula is valid only if the eight values we began with form the complete population. **If the values instead were a random sample drawn from some larger parent population, then we would have divided by 7 (which is $N-1$) instead of 8 (which is N) in the denominator of the last formula, and then the quantity thus obtained would be called the *sample* standard deviation.**

As a slightly more complicated real-life example, the average height for adult men in the United States is about 70 in, with a standard deviation of around 3 in. This means that most men (about 68 percent, assuming a normal distribution) have a height within 3 in of the mean (67–73 in) – one standard deviation – and almost all men (about 95%) have a height within 6 in of the mean (64–76 in) – two standard deviations. If the standard deviation were zero, then all men would be exactly 70 in. tall. If the standard deviation were 20 in, then men would have much more variable heights, with a typical range of about 50–90 in., and the bell curve would be more spread out. Three standard deviations account for 99.7 percent of the sample population being studied, assuming the distribution is normal (bell-shaped).

Calculating the Standard Error of the Mean

Simply divide the standard deviation by the square root of N (the number of data points used).

Standard error of mean versus standard deviation

They are not the same thing. Standard error is an estimate of how close to the population mean your sample mean is likely to be, whereas **standard deviation** is the degree to which individuals within the sample differ from the sample mean. Standard error should decrease with larger sample sizes, as the estimate of the population mean improves. Standard deviation will be unaffected by sample size.

Sample Problem:

Suppose you have two samples of 4 rats each. The lengths of the rats' tails in samples A and B are

A = 7.2, 7.0, 6.8, 7.0 cm

B = 3.6, 12.5, 3.3, 8.6 cm

a) Calculate the means for both samples.

b) Calculate the Standard Deviation for each sample.

c) Calculate the Standard Error of the Mean for each sample.

d) What do the SD and SEM tell you about the two samples that the mean alone does not? Explain.